STOCHASTIC DEFICIT MICRO IRRIGATION OPTIMIZATION

OPTIMISATION STOCHASTIQUE IRRIGATION DÉFICITAIRE MICRO

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ABSTRACT

A short term stochastic optimization-simulation model of deficit micro-irrigation, which distributes crops water stress over whole growing season, has been developed and tested in this study. The model, which is a nonlinear program with an economic objective function, considers interaction of stochastic rainfall and irrigation. It includes an eco-hydrologic-based simulation model that integrates an explicit stochastic analytical soil moisture model with the FAO crop yield model. Under some simplistic assumptions, analytical expressions have been derived for estimating expected value of crop yield and irrigation requirement volume along with assessing credibility of the assumptions made. While the developed explicit stochastic optimization model is of NLP form and showing convexity properties, it is computationally efficient comparing a similar implicit model which is time-consuming due to necessity of simulating the system for many realizations of rainfall events. Therefore, the model was effectively used in multi-crop situations and is extendable to be utilized in long-term irrigation planning models. The model was used in Dasht-e Abbas irrigation district of Karkheh basin in southwest of Iran, multi-crop realistic case. Results show that the proposed modeling approach with fast converging property is computationally efficient. It was also observed that for a multi-crop case with the same soil and climate conditions for all crops, three key factors, including potential crop yield, crop price and irrigation demand; chiefly participate in denoting the best deficit irrigation strategy under water shortage condition.

Résumé et conclusions

l'incertitude des précipitations, en tant que partie intégrante de la planification et l'ordonnancement irrigation déficitaire, doit être pris en compte dans l'irrigation déficitaire de modélisation. Des expressions analytiques pour les valeurs attendues du rendement et de volume exigence d'irrigation ont été obtenues et utilisées comme le modèle de simulation d'un modèle à court terme d'optimisation stochastique non linéaire pour la planification de l'irrigation déficitaire. Le modèle avec une fonction économique objective, qui considère l'interaction de la stochastique de pluie et d'irrigation, intègre les expressions dérivées d'analyse avec le modèle de la FAO le rendement des cultures. Le modèle a été résolu par SQP qui s'est avéré être

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efficiente de calcul par rapport à SDP et Monte Carlo en fonction des formulations utilisées dans la littérature qui deviennent inefficaces pour des problèmes de calcul multi-cultures.
Pour le cas multi-cultures de Dasht-e district d'irrigation de Abbas Karkheh bassin en Iran, il a été constaté dans la stratégie d'irrigation déficit qui baisse de 50% dans l'eau appliquée pour l'irrigation a conduit à seulement 20% de réduction de l'avantage net total (). Aussi dans un état extrême avec réduction de 80% dans l'eau d'irrigation, le bénéfice net total diminue de 50% par rapport à sa valeur maximale.
Le modèle proposé, dans lequel un modèle à base physique stochastique de l'irrigation déficitaire a été intégré dans un algorithme d'optimisation convergence rapide appropriés pour résoudre des problèmes multi-cultures, est avantageux par rapport à d'autres modèles présentés pour optimiser les calendriers d'irrigation dans l'incertitude. Toutefois, l'étude est la première étape dans l'application d'un célèbre modèle stochastique humidité du sol dans l'optimisation de l'irrigation déficitaire. À cet égard, l'élaboration d'un modèle analytique qui fournit de l'information probabiliste complète sur le rendement serait d'une grande importance. Explorer la possibilité d'utiliser plus détaillée des modèles de simulation stochastique de l'irrigation déficitaire fournissant pdf-s des variables aléatoires d'intérêt et aussi d'autres types de modélisation des stratégies de l'irrigation déficitaire (RDI et PRD) dans le cadre proposé de simulation-optimisation devraient être considérés pour de futures études .

INTRODUCTION

Deficit irrigation (DI) is known as a management measure toward solving water scarcity problem and improving water productivity. In other hands, rainfall and irrigation water (surface water resources) which play the most important role in supplying agricultural water requirement, are under uncertainty that needs to be taken into account. In this regard, stochastic dynamic programming (SDP) models have been developed to precisely survey irrigation planning under uncertainty for a single-crop farm [*Dudley et al.,* 1971a; *Dudley and Burt.,* 1973] and multiple crop farm by means of hybrid modeling techniques including: SDP - LP (Linear Programming) [*Dudley et al.*, 1976], SDP – DP (dynamic Programming) [*Vedula and Kumar,* 1996], SDP - NLP (Nonlinear Programming) [*Ghahreman and Sepaskha,* 2002], two-phase SDP [*Umamahesh and Sreenivasulu,* 1997]. These models constitute from three essential components: Monte Carlo technique for generating time series of stochastic processes, simulation of the main system and dynamic programming (DP). Consequently, in addition to the problem of curse of dimensionality of DP approach (limited to consideration of two or three state variables), utilizing Monte Carlo simulation, which is also a time consuming numerical method and needs approximation due to discretization of variables, leads to computational inefficiencies.

The above mentioned weaknesses were a starting point for an idea of developing explicit stochastic models in which an analytical simulation component was coupled with optimization techniques such as SDP [*Bras and Cordova,* 1981] or NLP [*Ganji et al.,* 2006]. While employing helpful idea of combining optimization techniques with analytical-based simulation models, these models have been limited to single-crop problems and either still using dynamic programming approach or including too simplifying assumptions to deal with stochastic nature of uncertainties.

More recently, significant analytical and realistic stochastic soil moisture model have been intensively under consideration [*Rodriguez-Iturbe and Porporato,* 2004]; however, it may not be easy to put it in use in an optimization modeling framework. Building upon the analytical model, this paper is going to presents a new stochastic optimization model for scheduling short-term deficit irrigation that while is efficient in dealing with medium and large scale problems, like multiple crop planning, considers realistic stochastic nature of rainfall in a precise daily time scale.

Model Formulation

Irrigation scheduling is generally defined as the problem of determining timing and amount of irrigation water to be supplied to a given crop area, in a given geographical region, during the growing season [*Rhenals and Bras,* 1981]. With recent advances in soil moisture monitoring systems and also modern irrigation technologies, DI can be implemented more precisely and effectively. A specific method of DI is called *deficit micro-irrigation* [*DeTar,* 2004]. By deficit micro-irrigation (DMI) we mean keeping relative soil moisture between rainfall events in a pre-specified level called soil moisture lower bound, which is below the point of incipient stomatal closure and above the wilting point. This form of DI distributes water stress throughout the whole growing season. A short term model of DMI scheduling for a multi-crop field is to be presented in this section.

Objective function

The model is a mathematical program with an economic objective function of net benefit obtained from agriculture over growing season as follows:

$$\max \quad f(\vec{s}_L) = \sum_c A_c \times \left(\overline{Y}_{a,c}(s_{L,c}) \times Pe_c - TC_c\right) \tag{1}$$

where f is objective function that depends on vector of decision variables ($\vec{s}_L$), soil moisture lower bounds of crops, where c is crop index and $s_{L,c}$ represents each component of the vector. $\overline{Y}_{a,c}$ is expected value (e.v.) of actual yield of crop c that as shown in following of paper is a function of soil moisture lower bound. A_c is cultivation area of crop c and Pe_c and TC_c respectively represent crop price per unit mass and crop total production cost per unit area. Note that the term within parentheses on right hand side of equation (1) represents net benefit per unit area resulting from production benefit (crop yield multiplied by its price) minus production cost.
Crop yield, whose mean appeared in Eq. (1), is a stochastic variable. In this study, founding on soil moisture dynamics, an analytical expression is presented to estimate expected value of crop yield. Other terms in the objective function, except $\vec{s}_L$, are supposed to be given.

Constraints

There are two main groups of constraints. The first group of constraints limits total irrigation volume as it must be less than total available water in the season:

$$\sum_c \overline{V}_c(s_{L,c}) \le TW \tag{2}$$

Where $\overline{V}_c(.)$ is e.v. of irrigation volume for crop c, which is a function of soil moisture lower bound, and TW is total volume of available water. The function relating $\overline{V}_c$ and $s_{L,c}$ has been introduced in the following. The second group of constraints limit actual yield of each crop to a minimum threshold ($Y_{\min,c}$),

$$Y_{\min,c} \le \overline{Y}_{a,c}. \tag{3}$$

Stochastic soil moisture model of deficit micro irrigation

A physically-based stochastic soil moisture model [*Rodriguez-Iturbe and Porporato,* 2004] has been used to derive analytical expressions for expected values of actual crop yield and probability density function of irrigation volume. Soil moisture dynamics, at daily time scale, is modeled by treating the active soil as a reservoir with an effective storage capacity that is intermittently filled by effective rainfall pulses of random depth. Soil water losses occur via evapotranspiration, deep infiltration, and surface runoff. Assuming vertically-averaged conditions the soil water balance dynamics can be expressed as:

$$nZ_r \frac{ds(t)}{dt} = R(t) + I(s(t)) - ET(s(t)) - LQ(s(t)) \qquad (4)$$

where n is the soil porosity, Z_r is the active soil depth, wherein most of the root is located, and s is the relative soil moisture in excess of its amount at wilting point with $s = 0$. The inputs to the soil moisture balance equation are effective rainfall, $R(t)$, and irrigation, $I(s(t))$, where effective rainfall is an instantaneous event occurring according to a marked Poisson process with parameters λ' (mean frequency of events) and α (mean intensity of each event value) and irrigation is modeled by means of keeping relative soil moisture between rainfall events in a pre-specified level called soil moisture lower bound, which is below the point of incipient stomatal closure and above wilting point [*Vico and Porporato,* 2010].

The main soil water losses include deep infiltration and runoff losses, $LQ(s(t))$, that in a simplified manner is assumed to take place instantaneously whenever soil moisture reaches the upper threshold s_U, and evapotranspiration, $ET(s(t))$ which is modeled through a piece-wise linear loss function as follows,

$$\rho(s) = \begin{cases} \eta \dfrac{s}{s^*}, & 0 \le s < s^* \\ \eta, & s^* \le s \le s_U . \end{cases} \qquad (5)$$

where $\eta = ET_p/(nZ_r)$ and ET_p is maximum rate of daily ET. Therefore the soil water loss function could be expressed only based on ET. Figure 1 shows a realization of such a stochastic soil moisture process.

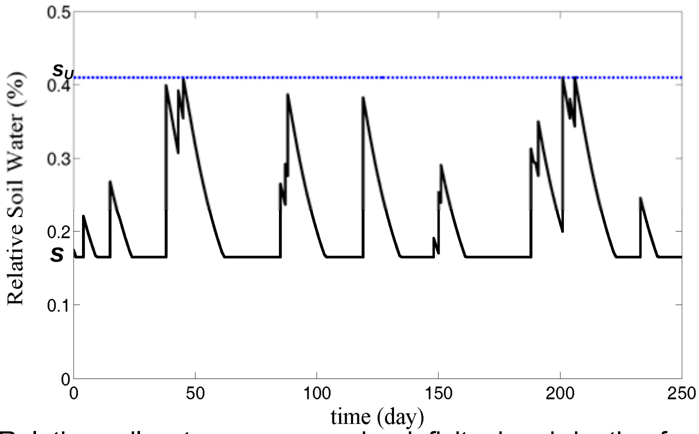

Figure 1. Relative soil water process under deficit micro-irrigation for loam soil with normalized upper bound of 0.41, close to normalized field capacity, and lower bound of 0.17 (Relative processus de l'eau du sol en conditions de déficit de micro-irrigation pour les sols limoneux avec normalisée limite supérieure de 0.41, proche de la capacité au champ normalize, et une limite inférieure de 0.17)

Rainfall randomness causes stochasticity in soil moisture process and its constituents. Consequently, Chapman-Kolomogorov equation for soil moisture process could be written by separation to two parts [*Vico and Porporato,* 2010]. The first part is for continuous distribution that is:

$$\frac{\partial}{\partial t} p_{dm}(s,t) = \frac{\partial}{\partial s}\left[\rho(s) p_{dm}(s,t)\right] - \lambda' p_{dm}(s,t) + \lambda' \int_{s_L}^{s} p_{dm}(u,t) f(s-u,u) du + \lambda' p_{0,dm} f(s-s_L, s_L) \tag{6}$$

And the second part for mass (atom) of probability at s_L as follows:

$$\frac{d}{dt} p_{0,dm}(t) = -\lambda' p_{0,dm}(t) + \rho(s_L) p_{dm}(s_L,t). \tag{7}$$

Under stochastic steady-state condition ($\partial p_{dm}(s,t)/\partial t \to 0$), pdf of soil moisture for DMI is resulted as:

$$p_{dm}(s) = \frac{C_{dm}}{\rho(s)} \exp\left(-\gamma s + \lambda' \int_{s_L}^{s} \frac{du}{\rho(u)}\right), \tag{8}$$

while atom of probability is:

$$p_{0,dm} = \frac{C_{dm}}{\lambda'} e^{-\gamma s_L}, \tag{9}$$

Where C_{dm} is the normalization constant such that $1 - p_{0,dm} = \int_{s_L}^{s_U} p_{dm}(u) du$.

Crop yield

Crop yield model of FAO [1979] states that decrease in cumulative actual evapotranspiration causes decrease in crop yield, as follows:

$$1 - \frac{Y_a}{Y_p} = k_y \times \left(1 - \frac{CET_a}{CET_p}\right) \tag{10}$$

where Y_a and Y_p are respectively actual and potential crop yields, k_y represents crop sensitivity to water stress, and CET_a and CET_p are cumulative actual and potential (maximum) ET over growing season, respectively. By means of piece-wise form of $\rho(s)$ and based on probability density function of soil moisture represented before, someone can derive an analytical equation relating e.v. of actual crop yield and other variables of interest, particularly the decision variable (s_L) as follows:

$$\begin{aligned}\overline{Y}_a = \overline{Y}_p \times \{1 - \overline{k}_y \times C_{dm} \times (& \frac{e^{-\gamma s_L}}{\lambda' s^*}(s^* - s_L) \\ & + \frac{s^*}{\eta}(\gamma s_L)^{-\lambda' s^*/\eta} \Gamma(\lambda' s^*/\eta)\left(P(\lambda' s^*/\eta, \gamma s^*) - P(\lambda' s^*/\eta, \gamma s_L)\right) \\ & - \frac{1}{\eta\gamma}(\gamma s_L)^{-\lambda' s^*/\eta} \Gamma(1 + \lambda' s^*/\eta)\left(P(1 + \lambda' s^*/\eta, \gamma s^*) - P(1 + \lambda' s^*/\eta, \gamma s_L)\right))\},\end{aligned} \tag{11}$$

Where $\Gamma(.)$ and $P(.)$ are respectively gamma and regularized incomplete gamma functions and C_{dm} is a normalizing factor.

Irrigation requirement volume

Taking *Vico and Porporato's* approach [2010] and assuming steady-state condition, e.v. of irrigation requirement volume with respect to s_L can be derived for DMI as:

$$\bar{V}_{dm} = nZ_r \rho(s_L) p_{0,dm} T_{seas} = \frac{nZ_r \eta T_{seas}}{s^* \lambda'} s_L e^{-\gamma s_L} C_{dm}(s_L) \quad (16)$$

Where T_{seas} is duration of growing season and other parameters are already defined.

One of the most popular and robust algorithms for solving NLPs is Sequential Quadratic Programming (SQP). SQP is based on solving a series of sub-problems that are Quadratic Programming (QP) approximation of original problem in each step. It was found while solving the above formulation via SQP that the model is fast converging. In most cases, the optimization procedure consumed less than 26 iterations to complete the task at most in 2 seconds for the case of 7 crops.

Results and Discussion

Dasht-e Abbas Irrigation District (DAID) with an area of 16500 hectares is located downstream of Karkheh Dam [*Mahab Ghodss Consulting Engineers,* 2001]. Among all irrigation districts in the region, DAID has rather proper water and soil resources. The climate is semi-arid with average annual rainfall of 280 mm and annual potential ET of 1625 mm. Based on analysis of daily rainfalls at Dasht-e Abbas climatology station, reduced rainfall frequency (λ') and its depth (α') are estimated as 0.088 day^{-1} and 1.12 cm, respectively. About 75% of cultivation is taken place during winter. Agronomic and economic properties of winter crops in DAID are presented in Table 1.

Table 1. Agronomic and economic properties of winter crops in Dasht-e Abbas Irrigation District (Agronomiques et des propriétés économiques des cultures d'hiver à Dasht-e Abbas Irrigation District) [Mahab Ghodss Consulting Engineers, 2001]

Crop	Type	Plant. Date	Area (ha)	ET_{max} (cm/day)	T_{seas}	Z_r (cm)	K_y	Max Yield (Kg/ha)	Price (Rials/Kg)	Total Cost (Rials/ha)
Wheat	winter	Nov	4125	0.26	200	100	1	6000	1700	1.39E+06
Barley	winter	Nov	2475	0.23	200	100	1	5000	1250	1.23E+06
Fava bean	winter	Nov	1320	0.20	175	50	0.85	2500	3200	1.66E+06
Eggplant	winter	Oct	990	0.18	130	70	1.05	25000	650	5.96E+06
Cucumber	winter	Nov	825	0.16	130	70	1.1	20000	700	5.89E+06
Tomato	winter	Jan	825	0.38	155	70	1.05	25000	550	6.59E+06
Sugar beet	winter	Nov	1650	0.28	205	70	0.8	50000	390	4.37E+06

As the region suffers from water shortage, deficit irrigation should be taken into consideration. The proposed DI optimization model has been tested and analyzed in DAID as a multi-crop case. Sensitivity analysis of total available water for irrigation (TW) which is available at beginning of growing season, as presented in Table 2, could appropriately demonstrate interaction between rainfall and irrigation in a multi-crop system. TW can take values ranging from zero to a maximum value, $TW_{\max}$, which is equal to sum of maximum irrigation requirement volumes of different crops, i.e. $TW_{\max} = \sum_c V_{dm,c}(s_{L,c} = s^*)$.

Table 2. Multi-crop model results and sensitivity analysis with respect to total available water (Résultats du modèle multi - cultures et l'analyse de sensibilité à l'égard de l'eau totale disponible)

Total Water (mcm)		7	11	14	18	22	25	29	32	36
TW / TW$_{max}$ (%)		20	30	40	50	60	70	80	90	100
Net Benefit (10^9 Rials)		49	60	69	76	83	90	96	101	105
NB / NB$_{max}$ (%)		47	58	66	73	79	86	91	96	100
s_L Optimum	Wheat	0.13	0.13	0.13	0.19	0.25	0.29	0.33	0.33	0.33
	Barley	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.24	0.33
	Fava bean	0.07	0.07	0.07	0.07	0.07	0.16	0.27	0.29	0.33
	Eggplant	0.27	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.33
	Cucumber	0.27	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.33
	Tomato	0.14	0.14	0.29	0.32	0.32	0.33	0.33	0.33	0.33
	Sugar beet	0.13	0.25	0.30	0.32	0.32	0.33	0.33	0.33	0.33

Table 2 presents the optimum solution of the soil moisture lower bound (s_L) for different crops and for different total irrigation water, wherein $s*$, the normalized relative soil moisture at stomatal closure point, equals 0.33 that is the maximum possible value for s_L. It is seen that 50% decrease of applied water for irrigation has resulted in only 20% reduction in total net benefit (NB). Also in an extreme condition with 80% reduction in irrigated water, total net benefit reduces by 50% compared to its maximum value.

It could be verified that optimum values of the soil moisture lower bound are increasing while the irrigation water increases. With $TW / TW_{\max}$ varying from 20% to 30%, s_L approaches its maximum value for eggplant and cucumber, while there is no water stress throughout the growing period. With increase of $TW / TW_{\max}$, other crops have also reached to this stress-less situation; first sugar beet, then tomato, next wheat, after that fava bean, and finally barley. This ranking of crops in reaching stress-less situation is affected by two important factors presented in Table 3. The first factor is an economic one that is product of the maximum yield and the price for each crop and the second one is the irrigation volume. While final ranking has been reported in column 6, the ranking with respect to each of the above factors has been respectively presented in columns 3 and 5. For the first three places, i.e. eggplant, cucumber and sugar beet, both criteria are important to the ranking; while in the next four ones, only economic factor is influential.

Table 3. Assessment of crop domination in total water sensitivity analysis (Évaluation de la domination des cultures dans l'analyse de l'eau sensibilité totale)

Crop	**Max Yield × Price**		**Irrigation Req.**		**Order in Irrigation Policy**
	Value (10^6 Rials / ha)	**Order**	**Value** (cm)	**Order**	
Wheat	1.02	5	33.1	5	5
Barley	0.63	7	27.3	4	7
Faba bean	0.80	6	22.7	3	6
Eggplant	1.63	2	12.6	2	1
Cucumber	1.40	3	10.2	1	1
Tomato	1.38	4	45.7	7	4
Sugar beet	1.95	1	40.4	6	3

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